Pre-Calculus CP 1 – Section 8.3/8.4 Notes Day 2 **Name:** KEY

Solving Systems using Inverse Matrices

Investigation: Using multiplication, write the left side of the matrix equation as a single matrix. Then equate corresponding entries of the matrices. What do you obtain?

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$$\begin{aligned} 5x - 4y &= 8 \\ x + 2y &= 6 \end{aligned}$$

Use what you just figured out to write this linear system as a matrix equation

$$\begin{aligned} 2x - y &= -4 \\ -4x + 9y &= 1 \end{aligned}$$

$$\underset{A}{\begin{bmatrix} 2 & -1 \\ -4 & 9 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} -4 \\ 1 \end{bmatrix}}$$

$$2 \times 2 \quad 2 \times 1 \quad = 2 \times 1$$

In the problem above, we have three matrices. Label the first one matrix A, the second matrix X, and the third matrix B.

If this were an algebraic equation, what would you do to solve for X?

$$A \cdot X = B$$

$$X = \frac{B}{A}$$

What can you do instead of dividing?

multiply both sides by $\frac{1}{A} = A^{-1}$

With matrices, we do not use division like we do with numbers. Another name for reciprocal is inverse, so since you would normally think of dividing as multiplying by the reciprocal, now with matrices think of it as multiplying by the inverse!!

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Using the matrices from above,

The solution of a Linear System that starts as AX=B is $X = A^{-1} \cdot B$

Ex. 1) Write and solve a matrix equation for the linear system:

a) $-3x+4y=5$
$2x-y=-10$

$$\overset{A}{\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 5 \\ 10 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$$

$x = -7, \ y = -2$

b) $-2x-5y=-19$
$3x+2y=1$

$$\overset{A}{\begin{bmatrix} -2 & -5 \\ 3 & 2 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} -19 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

$x = -3$
$y = 5$

Ex. 2) Three dimensions!

a) $2x+3y+z=-1$
$3x+3y+z=1$
$2x+4y+z=-2$

$$\underset{A}{\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

$x = 2$
$y = -1$
$z = -2$

b) $2x+3y-z=14$
$4x+5y+2z=34$
$-x+3y-4z=20$

$$\underset{A}{\begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 3 & -4 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 14 \\ 34 \\ 20 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \cdot B = \begin{bmatrix} -6 \\ 10 \\ 4 \end{bmatrix}$$

$x=-6$
$y=10$
$z=4$

Remember, this is just another way to solve systems of equations. You also know how to solve by substitution, elimination, and graphing.

HW: p. 609 #53, 54, 59, 61, 65, 69, 72 **and** p. 628 #25, 27, 30